

PLANCK SCALE EFFECTS AND THE SUPPRESSION OF POWER ON THE LARGE SCALES IN THE PRIMORDIAL SPECTRUM

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The enormous red-shifting of the modes during the inflationary epoch suggests that physics at the very high energy scales may modify the primordial perturbation spectrum. Therefore, the measurements of the anisotropies in the Cosmic Microwave Background (CMB) could provide us with clues to understanding physics beyond the Planck scale. In this proceeding, we study the Planck scale effects on the primordial spectrum in the power-law inflation using a model which preserves local Lorentz invariance. While our model reproduces the standard spectrum on small scales, it *naturally* predicts a suppression of power on the large scales—a feature that seems to be necessary to explain deficit of power in the lower multipoles of the CMB.

1. Introduction

In the inflationary scenario¹, the perturbations corresponding to comoving length scales of cosmological interest today would have emerged from quantum fluctuations at the beginning of inflation with physical wavelengths smaller than the Planck length. Hence, in principle, quantum gravitational effects should have left their signatures on the primordial spectrum. This opens up the interesting possibility of probing trans-Planckian (TP) physics using the CMB².

The first year results of WMAP³ data show that the power in the quadrupole and the octopole moments of the CMB are *lower than* as expected in the best fitting Λ CDM models. The deficit of power in the lower multipoles can not be explained within the context of the standard inflationary models (unless these models are fine-tuned⁴) and suggests a possible

signature of TP physics.

Most of the earlier efforts in incorporating TP physics into the standard field theory have involved models which *break* local Lorentz invariance². However, theoretically, there exists no *apriori* reason to believe that Lorentz invariance may be broken at the scales of inflation. More importantly, recent observations of synchrotron emission from the Crab nebula seem to suggest that Lorentz invariance may be preserved to very high energies⁵. In such a situation, in order to study the TP effects on the primordial perturbation spectrum, it becomes important that we also consider models which preserve Lorentz invariance *even as they contain a fundamental scale*. In this proceeding, we consider one such model, evaluate the resulting spectrum of density perturbations in power-law inflation and also discuss its implications for the CMB angular power spectrum.

2. The model and its application to power-law inflation

In the inflationary scenario, the primordial perturbation spectrum per logarithmic interval, viz. $[k^3 \mathcal{P}_\Psi(k)]$, is given by¹

$$\int_0^\infty d(\ln k) [k^3 \mathcal{P}_\Psi(k)] = G_0^+(\tilde{x}, \tilde{x}), \quad (1)$$

where $G_0^+(\tilde{x}, \tilde{x}')$ denotes the Wightman function corresponding to a massless and minimally coupled, quantum scalar field (say, $\hat{\Psi}$) evolving in the inflating background and the spectrum is to be evaluated at Hubble exit. Therefore, in order to understand the effects of Planck-scale physics on the perturbation spectrum, we need to understand as to how quantum gravitational effects will modify the propagator of a massless scalar field in the inflationary background.

Motivated by the Pauli-Villars regularization procedure, we assume that the massless Wightman function, viz. $G_0^+(\tilde{x}, \tilde{x}')$, is modified due to TP effects (in a locally Lorentz invariant manner) to⁷

$$G_M^+(\tilde{x}, \tilde{x}') = G_0^+(\tilde{x}, \tilde{x}') - G_{k_c}^+(\tilde{x}, \tilde{x}'), \quad (2)$$

where $G_{k_c}^+(\tilde{x}, \tilde{x}')$ is the Wightman function of a massive scalar field of mass k_c and k_c denotes the cut-off scale which we shall assume to be three to five orders of magnitude above the Hubble scale during inflation. Then, following Eq. (1), we can define the resulting modified perturbation spectrum,

viz. $[k^3 \mathcal{P}_\Psi(k)]_M$, as follows:

$$\int_0^\infty d(\ln k) [k^3 \mathcal{P}_\Psi(k)]_M = G_M^+ (\tilde{x}, \tilde{x}) = G_0^+ (\tilde{x}, \tilde{x}) - G_{k_c}^+ (\tilde{x}, \tilde{x}). \quad (3)$$

Let us now consider the massless and massive scalar fields to be propagating in a power-law inflationary background described by the line element

$$ds^2 = a^2(\eta) (d\eta^2 - dx^2), \quad (4)$$

where η is the conformal time, $a(\eta) = (-\mathcal{H}\eta)^{(\beta+1)}$ with $\beta \leq -2$ and \mathcal{H} denotes the energy scale associated with inflation. If we assume that both the fields are in the Bunch-Davies vacuum, then the modified power spectrum $[k^3 \mathcal{P}_\Psi(k)]_M$ can be expressed as

$$[k^3 \mathcal{P}_\Psi(k)]_M = \frac{k^3}{2\pi^2 a^2} (|\mu_k|^2 - |\bar{\mu}_k|^2), \quad (5)$$

where μ_k and $\bar{\mu}_k$ denote the normal modes of the massless and the massive fields satisfying the differential equations

$$\mu_k'' + [k^2 - a''/a] \mu_k = 0, \quad (6)$$

$$\bar{\mu}_k'' + [k^2 + (k_c a)^2 - a''/a] \bar{\mu}_k = 0, \quad (7)$$

respectively. On comparing Eqs. (1) and (5), it is clear that, in our model, the TP corrections to the standard spectrum arise as a result of the contribution due to the massive modes. Before we proceed further with the evaluation of the corrections, we need to stress the following point: In the standard inflationary scenario, it is well-known that the amplitude of the spectrum corresponding to the massive modes decays at super-Hubble scales. In our model, the TP corrections to the standard spectrum are due to the massive modes. Hence, within the standard inflationary picture, the amplitude of these corrections would be expected to decay at super-Hubble scales. However, as the massive modes we have considered are supposed to represent TP corrections to the standard massless modes, in what follows, we shall assume that the mechanism that ‘freezes’ the amplitude of the standard spectrum at super-Hubble scales will also ‘freeze’ the amplitude of the corrections at their value at Hubble exit.

The mode functions for the massless field in power-law inflation can be expressed in terms of Hankel functions and the standard power-spectrum, evaluated at Hubble exit, is given by¹

$$[k^3 \mathcal{P}_\Psi(k)] = C \left(\frac{\mathcal{H}^2}{2\pi^2} \right) \left(\frac{k}{\mathcal{H}} \right)^{2(\beta+2)}, \quad (8)$$

where C is a constant of order unity. Unlike the massless case, the exact solution to the massive modes $\bar{\mu}_k$ is not known in power-law inflation. However, it can be shown that, for $k_c \gg \mathcal{H}$, the WKB solutions are valid for all $(k\eta)$ over a range of values of β and k of our interest⁷. On using the WKB solutions, we obtain the modified power-spectrum (5) *at Hubble exit* to be

$$[k^3 \mathcal{P}_\Psi(k)]_M \simeq C \left(\frac{\mathcal{H}^2}{2\pi^2} \right) \left(\frac{k}{\mathcal{H}} \right)^{2(\beta+2)} \left[1 - \bar{C} \left(\frac{\mathcal{H}}{k_c} \right) \left(\frac{k}{\mathcal{H}} \right)^{(\beta+2)} \right], \quad (9)$$

where \bar{C} is another constant order unity.

The following points are note-worthy regarding the above result:

- (i) Fig. (1a) contains the plots of the modified and the standard spectrum. It is evident from the figure that the modified spectrum exhibits a suppression of power at the large length scales while it remains scale invariant at the small length scales.
- (ii) Naively, one would expect that TP effects will leave their imprints only at the ultra-violet end of the spectrum. However, we find that TP effects lead to a modification of the spectrum at the infra-red end. This can be attributed to the fact that the longer wavelength modes leave the Hubble radius at earlier epochs thereby carrying the signatures of the TP effects.
- (iii) The modified spectrum (9) has some similarities to the power spectrum that has been obtained recently in non-commutative inflation⁶.
- (iv) Though the modified spectrum we have obtained exhibits a suppression of power around the expected values of k , the extent of the suppression is far less than that is required to fit the CMB observations. In order to illustrate this feature, in Fig. (1b), we have plotted the relative power spectrum (i.e. the ratio of the modified spectrum to the standard spectrum) of our model and the fit to the WMAP data proposed by Contaldi et al.^a.

3. Discussion

In this proceeding, we have studied the TP effects on the spectrum of primordial perturbations in power-law inflation using an approach that preserves local Lorentz invariance. We assumed that the TP effects modify the standard propagator in a particular manner. We find that the resulting modified spectrum remains scale invariant at the ultra-violet end, but,

^aContaldi et al.⁴ proposed the following form for the primordial spectrum:

$[k^3 \mathcal{P}_\Psi(k)]_{CPKL} = A_s k^{(n_s-1)} \left[1 - \exp - (k/k_*)^\gamma \right]$, where A_s and n_s are the amplitude and index of the standard spectrum, $k_* \simeq 5 \times 10^{-4} \text{ Mpc}^{-1}$ and $\gamma \simeq 3.35$

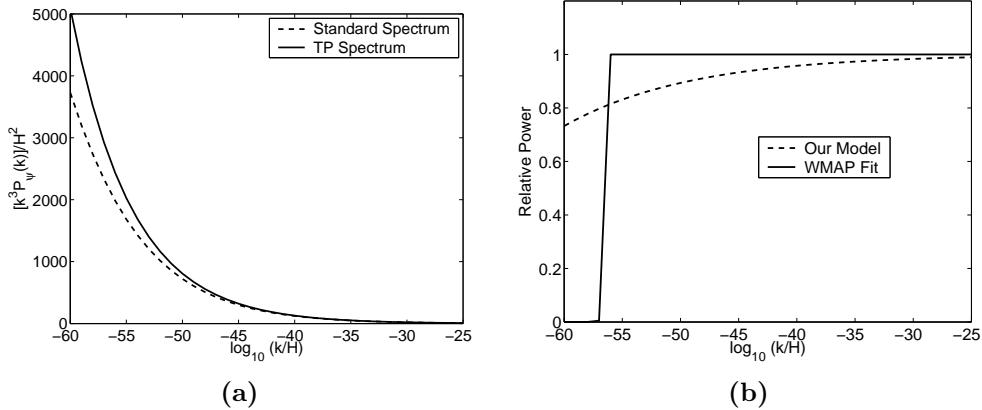


Figure 1. (a) Plots of the standard power spectrum (solid curve) and the modified power spectrum (dashed curve), normalized to \mathcal{H}^2 for $\beta = -2.04$. (b) Plots of the relative power of our model (dashed curve) and Contaldi et al.'s spectrum (solid curve). In both these plots, the dashed curve corresponds to $(\mathcal{H}/k_c) = 10^{-3}$ —value chosen so that the WKB approximation is valid around $k \sim 10^{-4} \text{ Mpc}^{-1}$ —and $\mathcal{H} = 10^{14} \text{ GeV} = 10^{52} \text{ Mpc}^{-1}$.

interestingly, it exhibits a suppression of power at the infra-red end—a feature that seems to be necessary to account for the deficit of power in the lower multipoles of the CMB^{3,4}. However, the amount of suppression predicted by our model in power-law inflation turns out to be *far less* than that seems to be required to fit the WMAP data. Nevertheless, the loss of power at large scales suggests that the power spectrum we have obtained may fit the WMAP data better than the standard Λ CDM model. It will be interesting to analyze the implications of our model for WMAP data in the context of slow-roll inflation.

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